

Name of Course	: CBCS B.Sc. (H) Mathematics
Unique Paper Code	: 32351401_OC
Name of Paper	: C8 Partial Differential Equations
Semester	: IV
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

- 1
- Derive the damped wave equation and telegraph equation of a string.
 - Reduce the equation $u_{xx} + xyu_{yy} = 0$, $x, y < 0$ to canonical form.
 - Prove the uniqueness of the solution of the initial boundary-value problem:

$$u_{tt} = 25 u_{xx}, \quad 0 < x < \pi, t > 0,$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq \pi,$$

$$u_t(x, 0) = g(x), \quad 0 \leq x \leq \pi,$$

$$u_x(0, t) = 0, \quad u_x(\pi, t) = 0, \quad t > 0$$
- 2
- Find the integral surfaces of the equation $uu_x + u_y = 1$ for the initial data $x(s, 0) = \frac{s^2}{2}$, $y(s, 0) = 2s$, $u(s, 0) = s$
 - Apply $e^u = v$ and then $v(x, y) = f(x) + g(y)$ to solve the equation $x^4 u_x^2 + y^2 u_y^2 = e^{-2u}$.
 - Use a separable solution $v(x, y) = g(x) + f(y)$ to solve the equation $v_x^2 + v_y^2 = v$.
- 3
- Find the characteristics and characteristic coordinates, and reduce the equation $u_{xx} - (2\cos x)u_{xy} + (1 + \cos^2 x)u_{yy} + u = 0$ to canonical form.
 - Determine the general solutions of the equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$
 - Transform the equation $u = -2u_{xx} - 9u_{xy} - 3u_{yy} - u_y$ to the form $v_{mn} = cv$, where $c = \text{constant}$, by introducing the

new variables $v = ue^{-(am+bn)}$, where 'a' and 'b' are undetermined coefficients.

- 4 (i) Determine the solution of the Cauchy problem given by

$$u_{tt} = 9u_{xx}, \quad u(x, 0) = x^3, u_t(x, 0) = x$$

- (ii) Determine the solution of the initial boundary value problem

$$u_{tt} = u_{xx}, \quad 0 < x < 1, t > 0$$

$$u(x, 0) = \sin \pi x, \quad 0 \leq x \leq 1$$

$$u_t(x, 0) = 0, \quad 0 \leq x \leq 1$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t \geq 0$$

- (iii) Determine the solution of characteristic initial value problem

$$xy^3u_{xx} = x^3 y u_{yy} + y^3u_x - x^3u_y,$$

$$u(x, y) = f(x), \quad y^2 + x^2 = 4, \quad 0 \leq x \leq 2$$

$$u(x, y) = g(y), \quad x = 0, \quad 0 \leq y \leq 2,$$

$$f(0) = g(2)$$

5. (i) Solve using method of separation of variables

$$u_{tt} = 9u_{xx}, \quad 0 < x < 2, t > 0$$

$$u(x, 0) = x(2-x), \quad 0 \leq x \leq 2$$

$$u_t(x, 0) = 0, \quad 0 \leq x \leq 2$$

$$u(0, t) = t, \quad u(2, t) = 0, \quad t > 0$$

- (ii) Determine the solution of

$$u_{tt} = u_{xx} + x^2, \quad 0 < x < 1, t > 0$$

$$u(x, 0) = 0, \quad 0 \leq x \leq 1$$

$$u_t(x, 0) = 0, \quad 0 \leq x \leq 1$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t \geq 0$$

- (iii) Solve the heat conduction problem

$$u_t = 9u_{xx}, \quad 0 < x < 1, t > 0$$

$$u(x, 0) = x^2(1-x), \quad 0 \leq x \leq 1$$

$$u(0, t) = 0, \quad t > 0$$

$$u(1, t) = 0, \quad t > 0$$

- 6 (i) Find the partial differential equation arising from the surface

$$2z = (ax + y)^2 + \beta$$

Find the solution of the Cauchy problem:

$$uu_x - uu_y = u^2 + (x + y)^2, \quad \text{with } u = 1 \text{ on } y = 0$$

(ii) Determine the solution of the initial boundary value problem

$$\begin{aligned} u_{tt} &= 9u_{xx}, \quad 0 < x < \infty, t > 0, \\ u(x, 0) &= 0, \quad 0 \leq x < \infty, \\ u_t(x, 0) &= x^3, \quad 0 \leq x < \infty, \\ u_x(0, t) &= 0, \quad t \geq 0 \end{aligned}$$

(iii) Solve the characteristic initial-value problem

$$xu_{xx} - x^3u_{yy} - u_x = 0, \quad x \neq 0,$$

$$u(x, y) = f(y) \text{ on } y - \frac{x^2}{2} = 1 \text{ for } 0 \leq y \leq 2,$$

$$u(x, y) = g(x) \text{ on } y + \frac{x^2}{2} = 3 \text{ for } 2 \leq y \leq 4,$$

with $f(2) = g(2)$.

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